ORBIT DETERMINATION AND ESTIMATION OF IMPACT PROBABILITY FOR NEAR EARTH OBJECTS

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INTRODUCTION

Near-Earth Objects (NEOs) are asteroids and comets that have been nudged by the gravitational attraction of nearby planets into orbits that allow them to enter the Earth's neighborhood. Most of the rocky asteroids we see today formed in the inner Solar System from debris leftover from the initial agglomeration of the inner planets, Mercury through Mars. Comets, on the other hand, are composed mostly of water ice with embedded dust particles, and originally formed in the cold outer planetary system, leftover bits from the formation of the giant outer planets, Jupiter through Neptune. Scientific interest in these objects is due largely to their status as the relatively unchanged remnant debris, the primitive, leftover building blocks of the solar system formation process. They offer clues to the chemical mixture of the primordial material from which the planets were formed some 4.6 billion years ago.

But these small bodies are interesting also because of the hazard they pose to the Earth. Even though the accretion phase of the Solar System ended long ago, as did the "heavy bombardment" phase, which produced the scars we see today on our Moon, Mercury, and other primitive bodies, the process of accretion and bombardment has not completely stopped. Today, the Earth is still accumulating interplanetary material at the rate of about one hundred tons per day, although most of it is in the form of tiny dust particles released by comets as their ices vaporize in the solar neighborhood. The vast majority of the larger interplanetary material that reaches the Earth's surface originates as fragments from the collision of asteroids eons ago. Larger pieces of debris hit the Earth less frequently simply because there are fewer of them. Van-sized asteroids impact the Earth approximately every few years, but typically disintegrate into small pieces before hitting the ground. Asteroids larger than about 50 meters, however, may well reach our surface largely in one piece, depending on their composition, and these impacts are estimated to

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occur approximately every few hundred years. An impat of this size would certainly cause a local disaster, and if it occurred in the ocean, it could produce a strong tidal waves that could inundate low lying coastal areas. On average, every few hundred thousand years or so, an asteroid larger than a kilometer will impact the Earth, and an impact of this size would almost certainly cause a global catastrophe. In this case, the impact debris would spread throughout the Earth's atmosphere so that plant life would suffer from acid rain, partial blocking of sunlight, and from the firestorms resulting from heated impact debris raining back down upon the Earth's surface. Even though asteroid and comet impacts of this sort are extremely infrequent, the enormous consequences of these events make it prudent to mount efforts to discover and study these objects, to characterize their sizes, compositions and structures and to keep an eye upon their future trajectories.

Because of the ongoing search efforts to find nearly all the large NEOs, objects will occasionally be found to be on very close Earth approaching trajectories. Great care must then be taken to verify any Earth collision predictions that are made. Given the extremely unlikely nature of such a collision, almost all of these predictions will turn out to be false alarms. However, if an object is verified to be on an Earth colliding trajectory, it seems likely that this collision possibility will be known several years prior to the actual event. Given several years warning time, existing technology could be used to deflect the threatening object away from Earth.

NEO SEARCH PROGRAMS

In 1990, the U.S. Congress directed NASA to organize a workshop to study ways of significantly increasing the rate of discovery of near-Earth asteroids (NEAs). The resulting workshop produced a proposal for an international program called the Spaceguard Survey, with the goal of discovering 90% or more of the NEAs larger than 1 km across within 10 years [Morrision 1992]. This size range was chosen because it poses the greatest impact hazard for our civilization on Earth. The estimated number of NEAs in this class is anywhere from 1500 to 2000, but only about 10% of this number have been discovered to date. Although the network of six 2.5-m telescopes proposed to implement the Spaceguard Survey was never funded, the accelerating pace of technology has enabled smaller and less expensive telescopes with modern detectors to at least partially achieve the discovery rate needed to complete the Spaceguard Survey.

Early efforts to discover NEOs relied upon photographic methods. Two plates or films of a given region of the sky would be taken many minutes apart, and then viewed through an instrument such as a special stereo viewing microscope. Any moving object such as an NEO would appear in a slightly different position on the two photographs, and, when viewed in stereo, the object would appear to "rise" above the background stars and galaxies, making it easy to find, but still a very labor-intensive operation.

Nowadays, NEO discovery teams use so-called charged couple devices (CCDs). These electronic detectors are not only more sensitive and accurate than the older photographic method, but they also record images digitally in arrays of picture elements (pixels), a form amenable to automated processing by a computer. A fairly common astronomical CCD detector might have dimensions of 2096×2096 pixels. The basic detection method is very similar to that used with the older photographic methods, but the detection is now automated using sophisticated computer-aided analyses of the CCD images. Separated by several minutes, three or more CCD images are taken of the same region of the sky. The algorithm then compares these images to see if any objects have systematically moved from one frame to the next. Once a moving object has been detected, the separation of the images in the successive frames, the direction of travel, and the object's brightness all are helpful in identifying how close the object is to the Earth, and its approximate size and orbital characteristics. In particular, an object that appears to be moving very rapidly from one frame to the next is almost certainly very close to the Earth.

Seven major NEO search programs are either in operation or in the planning stages. The oldest of these is the Spacewatch program, which has been operational since 1984 on Kitt Peak, near Tucson, Arizona. A second program, called the Near-Earth Asteroid Tracking (NEAT) program, is run out of the Jet Propulsion Laboratory using Air Force telescope facilities on Maui. Another search team that uses Air Force telescopes is the Lincoln Near-Earth Asteroid Research program (LINEAR) , a joint effort between the Air Force and the MIT Lincoln Laboratory, operating at Socorro, New Mexico. Using state-of-the-art detector technology, the LINEAR program is making about 70% of the current discoveries. Recently coming on line was the Lowell Observatory Near-Earth Object Search program (LONEOS) operating near Flagstaff, Arizona. A European cooperative NEO discovery effort between the Observatoire de la Côte d'Azur (OCA) in southern France and the Institute of Planetary Exploration (DLR) in Berlin, Germany, was initiated in October 1996. A second discovery effort near Tucson called the Catalina Sky Survey will soon be in operation, and in 2001, a Japanese search effort run by the Japan Spaceguard Association will begin operation.

In 1998, NASA made it a goal to help realize the aim of the Spaceguard Survey, namely to discover at least 90% of all NEOs whose diameters are larger than 1 kilometer within 10 years. As of the last half of 1998, the discovery rate of NEOs in this size range was about 70 per year, a factor of 6 less than the rate needed to achieve the Spaceguard goal [Harris & Morrison], but a significant increase over the rate only a few years earlier. With the promise of additional programs coming online and the bugs being worked out of some of the new programs, we may soon approach the discovery rate needed to achieve the Spaceguard aims.

NEO ASTROMETRY AND ORBIT CLASSIFICATION

Optical astrometric observations form the basis for orbit determination for asteroids and comets. To obtain an astrometric observation, a region of sky containing the object together with reference stars is imaged using either photographic or CCD techniques. The area of sky captured on the image must be sufficiently large to include at least a few stars from an astrometric catalog such as the Hubble Space Telescope Guide Star Catalog (GSC). The image positions of the object and catalog stars are measured and inverted using the known coordinates (right ascension and declination) of the reference stars to obtain the coordinates of the object: an optical astrometric observation consists of the right ascension and declination of the object, along with a time tag. Modern astrometric observations are typically accurate to better than one arc-second rms, with time tags accurate to 1 second or better. Errors introduced by the measurement process are usually no more than a few tenths of an arc-second, but a larger error is often introducted by inaccuracies in the coordinates of the reference stars. These are partly due to inaccuracies in estimates of proper motion of the stars, but mostly due to systematic errors introduced over entire regions of sky in the production of the catalog. Systematic star catalog errors can be as large as half an arc-second or more for some catalogs. An exception is the recently released Hipparcos catalog, a product of the European astrometry mission of the early 90s. Observations referenced to this catalog are typically accurate to a few tenths of an arc-second.

Once a handful of astrometric observations have been made for a newly discovered object, a preliminary orbit is computed using one of a variety of methods including ones suggested by C.F. Gauss in 1809 and by P.S. Laplace in 1780. These preliminary orbit methods consider only two-body motion; i.e., they do not include planetary perturbations. A review of these methods has been given by Marsden [1985]. Preliminary orbit solutions are very approximate, and, particularly if the object is a fast moving NEO, observers are quickly requested to provide so-called follow-up astrometric observations which could be used to improve the orbit solution before the object becomes lost. Once observations over a period of several days are available, the preliminary orbital elements are used to initiate a "definitive" orbit determination process that includes perturbations.

Asteroids and comets are often grouped into categories according to their orbital elements. The customary definition of a near-Earth object is an asteroid or comet with a perihelion distance less than 1.3 AU. Asteroids with semi-major axes larger than Earth's are referred to as Amors or Apollos, the former group having perihelia outside Earth's orbit $q > 1.017$ AU, and the latter group having perihelia inside 1.017 AU. Asteroids with semi-major axes less than Earth's, but aphelia outside Earth's perihelion (0.983 AU) are called Atens. A fourth group, analagous to the Atens, but with aphelia inside 0.983 AU, has no confirmed members yet, and has not been named. A more restrictive group called Potentially Hazardous Asteroids (PHAs) are asteroids whose orbit comes within 0.05 AU of the Earth's orbit, and whose absolute magnitude H is 22.0 or brighter (see next section). Figure 1 shows the orbit of the Apollo-type asteroid 1997 XF11, which makes one of the closest predicted approaches to the Earth. The line of nodes is the intersection of the orbit plane of the asteroid with the orbit plane of the Earth. Close approaches are possible when an object crosses its line of nodes at about the Earth's distance from the Sun.

As described earlier, optical astrometric observations form the basis for orbit determination of asteroids and comets. However, a second, highly accurate, form of astrometric observation is afforded by groundbased radar, which we use in our orbit solutions at JPL, when available. To date, radar astrometry has been obtained for 42 out of about 550 NEOs. This form of observation is made when a radio telescope is used to transmit an intense, focussed, coherent signal with known polarization state and time/frequency structure, and the echo delay and frequency is measured [Ostro 1994]. The echo time delay and Doppler frequency shift change continuously due to the relative motion of the target with respect to the radar, but these shifts can be largely removed using an *a priori* ephemeris for the object. The echo is dispersed in the time domain because of the finite size of the object (different parts of the object at different distances from the radar), and in the Doppler domain because the object rotates. Post processing of a number of transmit/receive cycles can be used to produce a normal points in delay and Doppler referred to the object's center of mass. Roundtrip delay can be measured to an accuracy of 1 microsecond or better, which corresponds to a range resolution of 150 m. Doppler frequency resolution can be as small as 0.1 Hz. The two radar telescopes used for the vast majority of radar observations of Solar System objects are the Arecibo and Goldstone instruments. The Arecibo system, which has been recently upgraded, works in the S-band at 2380 MHz, and is much more sensitive than Goldstone, but is limited to a 40-degree declination band because the dish is not steerable. The Goldstone antenna, operating in the X-band at 8510 MHz, is fully steerable, has access to all declinations north of -40°, and can track objects for longer periods of time. While the Goldstone antenna can observe a 1-km object out to a distance of about 0.10 AU, the Arecibo instrument can reach the same object at about 0.24 AU.

PHYSICAL CHARACTERIZATION OF ASTEROIDS AND COMETS

Just as astrometric observations are used to characterize the orbits of asteroids and comets, photometric observations can be used to characterize the physical properties of these objects. In particular, the size of an asteroid can be inferred from its absolute magnitude H, which is derived from a series of observations of the apparent magnitude normalized for the observing geometry. Specifically, H is defined to be the apparent magnitude of the object if it were at zero phase angle and at a distance of 1 AU from both the Sun and Earth. Using an assumed albedo A, we can estimate the approximate diameter D, in km, of the asteroid from the expression

$$\log D = 3.1236 - 0.2H - 0.5 \log A .$$

A mean albedo of 0.12 is typically assumed for poorly observed NEOs. Asteroids with absolute magnitudes of 18 or brighter will then have derived diameters of 1 km or larger. Other important physical characteristics can also be derived from the photometric data

[Cunningham]. From light curve data, the rotation period of the asteroid can be estimated, and often the pole position, and approximate shape of the asteroid can also be derived. Observations of occultations have also been used to determine the size and approximate shape of many asteroids.

To determine the composition of these objects, ground-based spectral analyses are conducted, and the asteroids grouped into spectral classes. We should be able to infer the elemental composition of asteroids by matching their spectra with the spectra of meteorite samples on Earth, which presumably comet from asteroid parent bodies in space. However, the connection between meteorite types (for which the composition is well known) and asteroid spectral class has not been firmly established, since the spectral type of the most common meteorite class (ordinary chondrites) is not particularly similar to the spectral features of the most common asteroid spectral class (type S) in the inner asteroid belt. This issue may soon be resolved in February 2000 when the Near-Earth Asteroid Rendezvous (NEAR) mission to asteroid Eros determines this object's elemental composition using a x-ray and gamma ray spectrometer.

Radar provides another powerful tool for characterizing NEOs. Size and roughness can be accurately determined, and, if the object gets near enough to the Earth, it can actually be imaged. Radar images of asteroid 4769 Castalia were taken during its approach to within 5.6 million km from Earth in 1989 (Hudson and Ostro, 1994), and asteroid 4769 Toutatis was imaged both in 1992 when it approached within 3.7 million km, and 1996 when it approached to within 5.3 million km (Ostro et al., 1995). Both Castalia (longest axis ~ 1.8 km) and Toutatis (longest axis ~ 4.6 km) showed evidence of being possible contact binary asteroids.

Asteroid structure and physical characteristics can also be derived from actual spacecraft images of these bodies. NASA's Galileo spacecraft imaged two asteroids, 951 Gaspra and 243 Ida, during flybys in October 1991 and August 1993, respectively while more recently, the Near-Earth Asteroid Rendezvous (NEAR) spacecraft imaged asteroid Mathilde and Eros in June 1997 and December 1998, respectively. The only comet nucleus to be imaged so far is that of comet Halley, which was visited by the European Space Agency's Giotto spacecraft in March 1986. It should be noted that comet Halley is in fact a near-Earth object.

To date, only two asteroids, Ida and Mathilde, have had their masses and densities determined using spacecraft data (Belton et al., 1995; Yeomans et al., 1997). While the bulk density of Ida was approximately 2.7 grams/cm^3, the bulk density of Mathilde turned out to be only 1.3 grams/cm^3 suggesting that this object is probably a collection of fragmented material with interior voids (rubble pile) rather than a monolithic slab of rock. We would expect that some asteroids are fragile ex-comets that have run out of volatiles, while others are almost solid iron, so that it seems likely that the range of asteroid compositions runs the spectrum from fragile rubble piles to slabs of metal. Once again spacecraft missions will be called upon to discern which types of asteroids fall into each category.

PRECISION ORBIT DETERMINATION

To be characterized, an orbit must first be parameterized, and many choices of parameterization are possible. We use the following six classical Keplerian orbital parameters: eccentricity e, perihelion distance q, time of perihelion passage, T_p, right ascension of the ascending node Ω, argument of perihelion ω, and inclination to the ecliptic i. The latter three parameters are referenced to the Earth mean orbital plane and equinox of J2000. Other orbit determination systems use slight variations of this set (e.g. semi-major axis a and mean anomaly M in place of q and T_p), or parameterizations such as equinoctial elements[ref]. The use of q instead of a avoids the singularity at parabolic orbits, which long period comets very nearly follow.

Active comets require additional orbital parameters to account for the non-gravitational perturbations caused by outgassing of the cometary nucleus. Sublimating frozen gases exit cometary vents at speeds of kilometers per second. and the reactive force on the motion of the nucleus center of mass can be substantial. Marsden et al. (1973) introduced a model for this acceleration based on the vaporization flux of water ice as a function of heliocentric distance. The nongravitational acceleration is modelled as an in-plane acceleration fixed relative to the radial direction and varying in magnitude with heliocentric distance. The two model parameters are A_1 and A_2, the radial and transverse components of the nongravitational acceleration at 1 AU. Yeomans and Chodas (1989) introduced an asymmetry into this model to account for a temporal lag in the strength of this acceleration, to account for the thermal lag in the heating of the nucleus.

The equations of motion are computed most easily in terms of the inertial-frame components of the heliocentric position vector $\mathbf{r}$ and velocity vector $\mathbf{v}$. Therefore, the first step in computing the motion of an object is the conversion from orbital elements to the position and velocity vectors at epoch. The Newtonian n-body equations of motion of the object are

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{\mu}{r^3}\mathbf{r} + \sum_{k=1}^{np} \mu_k \left(\frac{\mathbf{r}_k - \mathbf{r}}{\left|\mathbf{r}_k - \mathbf{r}\right|^3} - \frac{\mathbf{r}_k}{r_k^3} \right),$$

where $\mu = GM$ is the gravitational parameter of the Sun, np is the number of perturbing bodies, and μ_k and $\mathbf{r}_k$ are the gravitational parameter and heliocentric position vector of perturbing body k, respectively. Perturbations due to the nine planets and the Moon are included, with positions of these objects derived from the latest JPL planetary ephemeris (currently version DE405). Perturbations due to the three largest asteroids, Ceres, Pallas, and Vesta, are also often included, these taken from small-body ephemeris files developed at JPL. Added to the right hand side of the above equation are additional terms due to the $1/c^2$ relativistic perturbative accelerations, and, for comets, nongravitational accelerations. Direct analytical solution of the orbital equations of motion with all perturbations included is not possible. Instead, the equations are solved via numerical integration, a technique referred to as Cowell's method. We use a variable-order variable-stepsize self-starting Adams method developed at JPL, a code which features direct integration of the second order equation.

The basic orbit determination problem is to estimate the six orbital elements $\mathbf{x}_0$ at some epoch t_0, given a set of m astrometric measurements, $\mathbf{z}$. (The extended problem of including additional parameters such as A_1 and A_2 will not be discussed here.) A minimum of three astrometric observation pairs are needed to solve this problem, but since the observations inevitably contain errors, such a solution would be only approximate, especially if the observations are not widely spaced in time. Typically, we have dozens or hundreds of observations, and so $m >> 6$. The problem is therefore over-determined, and the method of least squares is used. The orbit determination problem is in fact the application for which Gauss invented this method in 1809. The method is often called batch least squares because the entire set of measurements at the various observation times are collected into the single vector $\mathbf{z}$. The order of the observations is unimportant. Gauss recognized that the optimal solution $\hat{\mathbf{x}}_0$ will minimize the sum of squares of the measurement residuals (the differences between the measured values and the values derived from the parameters).

Since the method of least squares assumes a linear relationship between measurements and solve-for parameters, while the orbit determination problem is very nonlinear, it is necessary to linearize the equations. This is accomplished by using an *a priori* estimate of the parameters $\overline{\mathbf{x}}_0$ as a reference and examining small variations about the reference, $\delta\mathbf{x} = \mathbf{x} - \overline{\mathbf{x}}_0$. The measurements must be modelled accurately as a function of the epoch reference state $\overline{\mathbf{x}}_0$ via the nonlinear measurement function $\mathbf{h}(\overline{\mathbf{x}}_0)$. Light time must be taken into account, and the *i*th measurement computed semianalytically by first integrating the object position to the time at which the light was emitted t_{ie}, determined iteratively, and then computing the value of the observation as a function of the object position $\mathbf{r}$ and velocity $\mathbf{v}$ at the emit time: $h_i(\overline{\mathbf{x}}_0) = h_i(\mathbf{r}(t_{ie}), \mathbf{v}(t_{ie}))$. Note that only radar Doppler measurements have a dependence on velocity. The linear least squares method is then applied by assuming that $\delta\mathbf{x}$ is small enough that it is related to the measurement residuals $\delta\mathbf{z} = \mathbf{z} - \mathbf{h}(\overline{\mathbf{x}}_0)$ via the linear equation

$$\delta\mathbf{z} \cong \mathbf{H}(\overline{\mathbf{x}}_0)\delta\mathbf{x} + \nu ,$$

where ν contains the measurement errors, which are assumed to be uncorrelated, and the measurement matrix $\mathbf{H}$ is the Jacobian of $\mathbf{h}$. In practice, the rows of $\mathbf{H}$ are computed sequentially, with each row given by the chain rule

$$\frac{\partial h_i}{\partial \mathbf{x}_0} = \frac{\partial h_i}{\partial \mathbf{r}(t_{ie})}\frac{\partial \mathbf{r}(t_{ie})}{\partial \mathbf{x}_0} + \frac{\partial h_i}{\partial \mathbf{v}(t_{ie})}\frac{\partial \mathbf{v}(t_{ie})}{\partial \mathbf{x}_0}.$$

The first partial in each of these terms is computed analytically as part of the measurement model. The second partial, called the transition or mapping matrix, can be computed in a variety of ways. If perturbations can be ignored, this partial can be computed analytically using the partials of two-body formulae, but this method looses accuracy if the object makes close approaches to planets. Alternatively, this partial can be evaluated via finite differences by numerically integrating orbits differing from the reference orbit in each component. In our software, we compute the transition matrix $\mathbf{Y}(t) \equiv \partial \mathbf{r}(t)/\partial \mathbf{x}_0$ via numerical integration of the so-called variational equations,

$$\ddot{\mathbf{Y}}(t) = \frac{\partial \ddot{\mathbf{r}}(t)}{\partial \mathbf{r}}\mathbf{Y}(t) + \frac{\partial \ddot{\mathbf{r}}(t)}{\partial \mathbf{v}}\dot{\mathbf{Y}}(t),$$

where the partials of $\ddot{\mathbf{r}}(t)$ are computed analytically. The variational equations are numerically integrated at the same time as the equations of motion.

The weighted least squares procedure minimizes the weighted sum of squares of the residuals, $\left\|\mathbf{W}^{1/2}(\mathbf{H}\delta\mathbf{x} - \delta\mathbf{z})\right\|^2$, where $\mathbf{W}$ is a diagonal matrix with the squares of the measurement weights on the diagonal. Measurement weights are the inverses of the assumed accuracies of the measurements. The solution to the weighted least squares problem is given by the normal equations,

$$\delta\hat{\mathbf{x}} = \mathbf{\Lambda}^{-1}\mathbf{H}^T\mathbf{W}\delta\mathbf{z},$$

where $\mathbf{\Lambda}$ is the normal matrix, also called the information matrix, given by

$$\mathbf{\Lambda} = \mathbf{H}^T\mathbf{W}\mathbf{H}.$$

The full nonlinear estimate is then given by $\hat{\mathbf{x}} = \bar{\mathbf{x}} + \delta\hat{\mathbf{x}}$, and the covariance matrix describing the accuracy of the estimate is given by $\mathbf{P}_0 = \mathbf{\Lambda}^{-1}$.

Rather than using the normal equations to solve the orbit determination probelm, we use a numerically more stable procedure called the square root information filter [Bierman]. Instead of forming the normal matrix for the problem, this method uses the Householder transformations to reduce the problem to a minimization of the quantity $\left\|\mathbf{R}\delta\mathbf{x} - \mathbf{z}'\right\|^2$, where $\mathbf{R}$ is an upper triangular matrix referred to as the square root information matrix; $\mathbf{R}$ and $\mathbf{z}'$ satisfy

$$\mathbf{T}\left[\mathbf{W}^{1/2}\mathbf{H} \quad \mathbf{W}^{1/2}\delta\mathbf{z}\right] = \begin{bmatrix} \mathbf{R} & \mathbf{z}' \\ \mathbf{0} & \mathbf{e} \end{bmatrix},$$

where $\mathbf{T}$ is a product of Householder orthogonal transformations and $\mathbf{e}$ is a vector of transformed residuals. Since $\mathbf{R}$ is square, the solution to the least squares problem is simply

$$\delta\hat{\mathbf{x}} = \mathbf{R}^{-1}\delta\mathbf{z}'$$

and the covariance matrix of the solution is computed as $\mathbf{P}_0 = \mathbf{R}^{-1}\left(\mathbf{R}^{-1}\right)^T$.

In practice, this process of differential correction must be repeated until the corrections $\delta\hat{\mathbf{x}}$ become negligible. Another check on convergence is that the residuals predicted linearly from the old solution match the actual residuals from the new solution.

ACCURACY OF THE ORBIT SOLUTION

An orbit is never known exactly; it always contains some level of uncertainty, which is quantified in the covariance matrxi $\mathbf{P}_0$. A primary determinant of the solution accuracy is the set of weights used to form $\mathbf{W}$. Primary determinants of the solution accuracy are the time span over which the object was observed (referred to as the "data arc"), the number and accuracy of the observations, and whether or not radar observations were used in the orbit solution. The data arc is often characterized by the number of oppositions during which the object was observed, where an opposition is centered on a passage by the Earth-Sun line. (For comets, we also speak of the number of "apparitions", where a new apparition starts when the comet passes aphelion.) If an asteroid has been observed over several oppositions, its orbit is typically quite well determined, and its position can be predicted ten years into the future with high accuracy, usually better a few arc-seconds on the plane of sky. Asteroids with such "secure" orbits are eligible for receiving an official number from the International Astronomical Union (IAU). The tally of such numbered asteroids is just now reaching 10,000.

Asteroids with data arcs of fewer than three oppositions have less well-determined orbits, and are therefore generally not numberable unless they have been observed with radar. Single-apparition asteroids have only moderately well-determined orbits, at best. Still, if the data arc covers many months, reasonably accurate predictions can be made several decades into the future. But, if the data arc is less than a couple months, the orbit accuracy deteriorates, and predictions beyond a decade or so are very uncertain. If the data arc is less than a week or so, predictions may not be accurate enough to ensure recovery during the next opposition, and the object may become lost. This is why follow-up observations in the week or so after discovery are so important.

It often happens that after an asteroid has been observed for a week or more, it can be linked to an asteroid discovered earlier but subsequently lost. Such an identification of a newly discovered object with a previously observed lost asteroid immediately extends the data arc to an earlier opposition, and improves the orbit quality dramatically. In other cases, if a newly discovered object has been observed for a month or so, predictions can be made for earlier oppositions in hopes of locating the previously undetected asteroid in photographic archives. Such a pre-discovery detection was made for asteroid 1997 XF11 in March 1998, when the asteroid was detected on photographic plates taken 8 years earlier. The "precovery" data extended the data arc dramatically and improved the orbit accuracy of this potentially hazardous object by an order of magnitude.

PREDICTING CLOSE APPROACHS AND IMPACT PROBABILITIES

Once the orbit for a NEO has been determined, we can determine its future close approaches to the Earth by numerically integrating its position forward in time and monitoring the Earth-object distance. If the NEO has a secure orbit, we integrate as far forward as the year 2200. Objects with less secure orbits are integrated forward over shorter time spans, according to the data arc. The step size of the numerical integrator is automatically so as to maintain a local velocity error of less than 10^{-13} AU per day. Perturbations by the Earth and Moon are considered separately, rather than treating their combined masses as being located at their barycenter. Perturbations by the three large asteroids Ceres, Pallas, and Vesta are included, and the general relativistic equations of motion are employed. When the integrator senses a planetary close approach, it interpolates within the trajectory to find the nominal close approach time to the one minute level, or better. (The actual uncertainty in the close approach time may be larger, due to the uncertainty in the object's initial orbital elements.) Close approaches to all perturbing bodies are detected, but only close approaches to the Earth are considered here.

For asteroids and comets with reasonably secure orbits, Table 2 lists the predicted Earth close approaches of PHAs to within 0.03 AU over the next century. The closest predicted Earth approach is that of asteroid 2340 Hathor in the year 2086, the miss distance being only 0.0057 AU. The 2028 encounter of 1997 XF11 is actually third closest, at 0.0064 AU. In addition to the close approach date and miss distance in both astronomical units and lunar distances (LD), the table lists the asteroid's absolute magnitude H, which is an indicator of the objects' size, and the final column gives the approximate number of sigmas required before the uncertainty ellipse would touch the Earth disk in the target plane. All numbers in this column are larger than a hundred, indicating that none of these close approaches will result in an impact.

In addition to the nominal miss distance of future close approaches, the uncertainties in the close approach circumstances must also be considered. The probability that a NEO will hit the Earth during one of the predicted close approaches is a function of the position and velocity uncertainties during the encounter. These uncertainties can be quickly approximated using a linear covariance procedure which maps orbit uncertainties at a contemporary epoch to position and velocity uncertainties at the predicted time of close approach. The mapping is accomplished by computing the mapping matrix **Y** at the close approach time t_{CA}. As before, the mapping matrix is computed via numerical integration of the variational equations.

We use two methods to estimate the impact probability. In the first method, described in [Chodas 1993 and Yeomans & Chodas 1994], we form the target plane as the plane perpendicular to the velocity vector of the object relative to the Earth, and compute the position uncertainty ellipsoid at the moment of close approach. This error ellipsoid is a representation of the scale and spatial orientation of a three-dimensional Gaussian probability density function. This ellipsoid is then geometrically projected into the target plane, forming an uncertainty ellipse. Since, to first order, all points within the ellipsoid cross the target plane orthogonally, the projection of the ellipsoid removes the element of time from the impact problem. Essentially, the uncertainty ellipse represents the marginal Gaussian probability density function describing the probability that the object will at some time during the encounter pass through a given point on the target plane. To first order, the figure of the Earth projects to a disk in this plane, and the probability of impact is computed by integrating the marginal probability density function over the Earth disk.

The position uncertainty in the target plane at close approach are approximated by linearly mapping the orbit uncertainties at epoch using the mapping matrix **Y**. Define the target plane frame with z-axis along the relative velocity vector, and x-axis along the minus relative position vector; then the x-y plane is the target plane. If **C** denotes the matrix representing the rotation from the inertial to the target plane frame, then a variation in epoch elements $\delta\mathbf{x}_0$ map to position variations in the target plane, $\delta\mathbf{r}_{tp}$, by

$$\delta\mathbf{r}_{tp} = (\mathbf{C}\mathbf{Y})\,\delta\mathbf{x}_0 .$$

The square root covariance of the position uncertainty in the target plane frame is therefore simply $\mathbf{S}_{tp} = (\mathbf{CY})\,\mathbf{R}^{-1}$, and the covariance given by $\mathbf{P}_{tp} = \mathbf{S}_{tp}\mathbf{S}_{tp}^{\;T}$. The projection of three-dimensional position uncertainty into the target plane is accomplished by simply deleting the third row and column of $\mathbf{P}_{tp}$. The probability of impact is then approximated by integrating the resulting two-dimensional Gaussian probability density over the Earth disk. For this integration , we use an efficient semi-analytic technique developed in the context of planetary quarantine and impact avoidance for spacecraft carrying nuclear materials [Michel 1977]. The uncertainty in the third component of $\mathbf{P}_{tp}$ is used for determining the uncertainty in the time of close approach by simply dividing it by the relative velocity.

Recently we have adopted a second linear method for estimating impact probability, which uses the so-called b-plane, or "impact" plane, the plane perpendicular to the incoming asymptote of the hyperbolic geocentric trajectory, and considers the asymptotic state of the object. This method is less susceptible to the problem of differential perturbations over the position uncertainty ellipsoid. The standard set of b-plane elements uses the geocentric position **b** of the intercept of the incoming asymptote with the impact plane, which is equivalent to the close approach position vector if the Earth were massless. We use a variation of the standard approach, by defining a scaled impact parameter $\mathbf{b}_p$ given by

$$\mathbf{b}_p = \frac{1}{v_p} \mathbf{s} \times \mathbf{h},$$

where **s** is the unit vector along the incoming asymptote, $\mathbf{h} = \mathbf{r} \times \mathbf{v}$ is the specific angular momentum vector, and v_p is the velocity at the planet's surface, given by

$$v_p = \sqrt{\frac{2\mu}{r_p} - \frac{\mu}{a}},$$

where r_p is the radius of the planet, μ is the gravitational parameter of the planet, and a is the semi-major axis of the object's two-body trajectory relative to the planet. Because of our use of v_p instead of the hyperbolic excessive velocity v_∞, the figure of the Earth in the impact plane is a disk of radius r_p, not the capture radius r_c. The square root covariance of the uncertainty in the scaled b-plane elements $\mathbf{x}_{sb}$ is given by

$$\mathbf{S}_{sb} = \left(\frac{\partial \mathbf{x}_{sb}}{\partial \mathbf{r}} \mathbf{Y} + \frac{\partial \mathbf{x}_{sb}}{\partial \mathbf{v}} \dot{\mathbf{Y}} \right) \mathbf{R}.$$

The upper left 2×2 partition of $\mathbf{P}_{sb} = \mathbf{S}_{sb} \mathbf{S}_{sb}^T$ describes the uncertainty in $\mathbf{b}_p$, and the probability of impact is computed in the same way as in the first method, by integrating this marginal probability density function over the Earth disk. The third component of our scaled b-plane elements is the linearized time of flight, effectively the close approach time for the rectilinear approach trajectory with the same energy, and its uncertainty is also computed.

These linear covariance methods for computing uncertainties can break down if the uncertainties grow too large, e.g. if the prediction period is long, or the object makes repeated close approaches to a planet. As the uncertainties increase, the linearity assumption becomes less and less tenable. For these cases, the semilinear confidence boundary method of Milani and Valsecchi [ref] can be used to determine a more precise confidence boundaries in the target plane. Basically, this method spreads many dozens of test points around the uncertainty ellipse of the linear method, linearly maps each of these back to variations in epoch elements, and then integrates each forward using the fully nonlinear equations of motion to find where they intersect the target plane. The process essentially warps the error ellipse to account for the non-linearities of the mapping to the target plane.

A more straightforward nonlinear method uses a Monte Carlo approach [Muinonen and Bowell], [Chodas&Yeomans 1994 (SL9)]. The six-dimensional ellipsoid representing the uncertainty of the orbital elements at epoch is populated with thousands of random test points to obtain an ensemble of initial conditions consistent with the 6×6 covariance matrix of the orbit solution. The test points are then all integrated forward using the fully nonlinear equations of motion, to the close approach in question, and a statstical analysis is performed on the resulting ensemble of close approaches. Muinonen then computes the probability of impact of the ensemble using an extrapolation method [ref]. The Monte Carlo method has the advantage of being fully nonlinear, but it requires a large amount of computation.

THE CASE OF ASTEROID 1997 XF11

Asteroid 1997 XF11 received much notoriety in March 1998, when for a time, orbit solutions indicated that it would make a remarkably close approach to the Earth. The asteroid had been discovered on December 6, 1997 by Jim Scotti using the Spacewatch Telescope on Kitt Peak, and had been placed on the Minor Planet Center's list of Potentially Hazardous Asteroids soon afterwards. After a month, its orbit was well enough determined for the Center to predict that the asteroid would pass relatively close to Earth on October 28, 2028: within a million kilometers. The asteroid was well observed for another month, but then went unobserved for four weeks. When Peter Shelus at the McDonald Observatory in Texas picked it up again on the nights of March 3 and 4, his four observations extended the data arc significantly, to 88 days, and changed the orbit solution significantly too. The new miss distance in 2028 had shrunk to less than a quarter of a lunar distance, and possibly even smaller, making it easily the closest-ever predicted close approach of an asteroid to the Earth. The fairly large size of the object, probably over a kilometer across, also made the object notable, and it would be fairly bright if it did indeed come as close to Earth as predicted. On March 11, Brian Marsden, director of the Minor Planet Center, announced the predicted extreme close approach of 1997 XF11 in 2028 in an IAU Circular, adding that passage within one lunar distance was "virtually certain". In an accompanying press statement, Marsden stated "The chance of an actual collision is small, but one is not entirely out of the question."

But, in fact, a complete analysis of the observations available on March 11, using the techniques outlined in this paper, shows that the probability of impact in 2028 was very, very small, essentially zero. Figure 2 shows the 3-sigma position uncertainty ellipse in the target plane at closest approach (sigma denotes the standard deviation). The ellipse is extremely elongated, about 2.8 million kilometers long, but only 2,500 km wide. The extreme length of the ellipse is due to the fact that the position uncertainty along the orbit grows linearly with time over the 30-year prediction period, while uncertainties perpendicular to the orbit vary only periodically. (It is interesting to note that the 30-year projection into the future spans over 17 revolutions of the asteroid about the Sun.) Since the ellipse extends well beyond the Moon's orbit, passage outside one lunar distance is very possible. The great length of the ellipse in Figure 2 makes it difficult to predict a precise miss distance, since passage virtually anywhere within the ellipse is possible, according the observations. The narrow width of the ellipse, however, allows a fairly precise determination of the *minimum possible miss distance*, about 28,000 km. Figure 3 shows a close up of the region of the target plane near the Earth. The ellipse would have to be enlarged to about the 55-sigma level before it would graze the Earth.

As it turned out, on March 12, Ken Lawrence and Eleanor Helin, both of JPL, found four pre-discovery images of the asteroid, taken in 1990. These greatly extended the data arc to 8 years, strengthening the orbital solution. The predicted close approach in 2028 moved out to a rather unremarkable 980,000 km, while the uncertainty ellipse shrank by over an order of magnitude. Figure 4 shows the error ellipse for the orbit solution including the 1990 observations. While the new observations moved the miss distance to a comfortable distance, they were not needed to rule out the possibility of collision in 2028.

WARNING TIME FOR AN OBJECT ON AN EARTH COLLISION COURSE

If an object were discovered to be on a collision course with Earth, how long before impact could we predict the event? The most important factor in this "warning" time is how long before impact the object is discovered. Of course, if the object is not detected at all while in space, the warning time is essentially zero. If the object is detected just hours or a few days before impact, the limiting factor would be how quickly the discovery could be reported and confirmed, and a few follow-up observations made. Even the earliest orbit determination solutions would likely indicate impact because the object would be quite close and the astrometry therefore quite powerful in determining the orbit. Radar could hardly be used in this scenario because the observations require at least a few days preparation. On the other hand, if the incoming object is discovered a few weeks to a month before impact, impact could likely be predicted only a week or so after discovery, and radar observations could be taken to precisely determine the orbit, and at least pinpoint the impact location and time.

For objects discovered many months, years, or decades before impact, the warning time can be much shorter than one would expect. Early orbit solutions would establish the likelihood of an extremely close approach, but the orbit uncertainties could easily be too large to distinguish between an impact and a near miss. A key factor is the number of oppositions in the object's data arc. If the dangerous close approach occurs during the object's discovery opposition, it may not be possible to reliably predict a hit or a miss much more than a few months ahead of the event. The same limit applies to long period comets such as Hale-Bopp and Hyakutake, which would not have not been observed previously in recorded history. In the more hopeful scenario of an asteroid discovered during one opposition, but not making a close approach until a later opposition, a definitive impact prediction would likely have to wait until the asteroid was observed during a second opposition, or pre-discovery observations were found, as with 1997 XF11. As soon as the asteroid's data arc spans multiple oppositions, the orbit becomes much better determined, and a definitive impact prediction could be made, even if the impact is many decades in the future. Radar observations could play an important role in improving the orbit accuracy, but their effect is secondary compared to the length of the data arc.

CONCLUSION

Near-Earth Objects (NEOs) are, at the same time, scientifically important and potentially hazardous to the Earth. In order to evaluate the threat these objects pose, the first order of business is to search for and discover the vast majority of the population of large NEOs which has not yet been detected. Our growing set of NEOs must then be observed astrometrically over a period of months and years, and increasingly accurate orbits determined for them. Each object's motion must be numerically integrated forward in time to determine how close it might come to the Earth. Close approach uncertainties must be carefully examined, and, if any extremely close approaches are possible, robust impact probabilities must be accurately computed. If a non-neglible impact probability is found for any object, impact mitigation methods must be developed, and this will require that the object's physical characteristics be well understood. This knowledge can be acquired through a combination of extensive ground-based photometric and spectroscopic observations together with space missions for recconnaissance and mitigation technology development. If an object is on a collision course with Earth, it is critical to know this fact as early as possible, and the most important determining factor in this lead time is how early the object was discovered.

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